Ladder Pricing – A New Form of Wholesale Price Discrimination

Ian McQuin Dobbs

Newcastle University Business School

5, Barrack Road

Newcastle upon Tyne NE1 4SE, UK.

ianmdobbs@btinternet.com

Abstract

Wholesale 'ladder pricing' involves setting the wholesale price a retailer faces as a nonlinear (generally increasing) function of the price chosen by that retailer. The special case where the ladder pricing contract is linear is shown to be equivalent to a form of revenue sharing. Optimal profit maximizing ladder pricing/revenue sharing is examined, given that retailers are privately informed of their demands and costs, and have control over whether they participate, and if so, what retail price they set. The solution is compared with the alternative of wholesale quantity discounting as relative level of retailer demand/cost heterogeneity is varied; ladder pricing/revenue sharing tends to outperform quantity discounting by an increasing amount the greater retailer demand heterogeneity is relative to cost heterogeneity. Ladder pricing has recently been implemented for 08 and related calls in UK Telecoms and has been subject to extended legal dispute; the case and issues involved are discussed.

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1. **Introduction**

This paper seeks to bring attention to a type of wholesale pricing contract hereafter referred to as 'ladder' or 'tiered' wholesale pricing. This form of wholesale pricing contract does not seem to have been discussed in the wholesale pricing literature, so it seems useful to explain its form, examine its scope, and make comparisons with more familiar forms of wholesale contract. A 'ladder' wholesale pricing contract is a contract in which the wholesale price levied for each participating retailer is contingent on the downstream price that retailer subsequently chooses. Recent examples in UK telecoms practice have featured linear and/or non-linear, continuous and/or step functions in which the wholesale price is set as an increasing function of retail price, hence the terms 'ladder' or 'tiered' for the wholesale pricing contract.

The paper (i) examines the extent to which ladder pricing can help to coordinate[1] supply channels, (ii) discusses the linkage between ladder pricing and revenue sharing and shows that a linear ladder pricing contract is, in some circumstances, equivalent to revenue sharing in conjunction with uniform pricing, (iii) examines the relative performance of ladder pricing *vis a vis* quantity discounting, showing how relative performance is positively related to the extent of heterogeneity in retailer demands *vis a vis* their costs, (iv) discusses the pros and cons of choosing a step function versus a continuous function for the ladder pricing contract and (v) discusses recent and possible legal and regulatory attitudes to this form of pricing.

UK Telecom fixed line network providers have recently introduced ladder pricing contracts as a means of revenue sharing for 080/0845/0870 and related call numbers (often referred to as NGNs, or non-geographic numbers). These numbers are used in the UK by organizations, both public and private, for help lines etc. The 080 number ranges have been typically free to use by those who make a fixed line call. By contrast, mobile network operators (MNOs) have often made significant charges for such calls. Such calls to service providers are typically terminated by the fixed line networks involved, and they have tried to implement wholesale ladder pricing in order to 'revenue share' with the MNOs on these number ranges. The mobile network operators appealed the imposition of such wholesale pricing, and the issue has gone before the regulator Ofcom and thence to the Competition Appeal Tribunal, the Court of Appeal, and then the Supreme Court. The Supreme Court made a final ruling (Supreme Court, 2014) in July 2014 that such a form of pricing should indeed be permitted. The 'economics' of this case revolved around whether the proposed wholesale charges actually gave mobile operators an incentive to reduce their retail prices for these ‘social’ calls.[2] Clearly, if the wholesale charge rises in some sense ‘fast enough’ with the choice of retail price, retailers will have an incentive to reduce prices, and that is likely to be beneficial to final consumers.

The wholesale pricing problem examined is one in which a profit maximizing monopoly wholesaler faces a significant population of potential downstream profit maximizing retailers, knows the distribution function for their type, a joint distribution including both demand and cost characteristics, but is unable to distinguish them by type. The wholesaler posts a contract, and the retailers then choose whether to participate, and if so, what price they will charge at the retail level. Because of the timing of decisions, this format is equivalent to a Stackelberg game in which the wholesaler is the leader and retailers are followers, an important class of game used in past literature to model entry decisions by firms (e.g. Dixit, 1982).

In this setting, if there is just one retailer, it is well known that quantity discounting schemes, including a simple two part tariff, can be used to fully coordinate the distribution channel (e.g. Moorthy, 1987) and may perform well even in more complex settings (e.g. Raut *et al*., 2008). However, when there are multiple heterogeneous retailers, and participation is a retailer choice variable, a two part tariff cannot be used to coordinate the distribution channel and the wholesaler can do better profit wise by deviating from full coordination (Ingene and Parry, 1995; Ingene *et al.*, 2012). Basically, when constrained to set the same fixed charge to all participating retailers, a profit maximizing wholesaler has an incentive to set a higher than marginal cost price, and this implies a reduction in overall channel profitability. Using a more general quantity discounting contract will improve performance, but there is still a fall away from full coordination joint profitability. If the wholesaler is able to monitor ex post the prices the participating retailers choose to set, then setting ex ante a standard contract which ties wholesale price charged to the price chosen downstream by the retailer can usually be designed to increase wholesaler profitability. This contract typically features a higher wholesale price, the higher the price subsequently chosen by a retailer, hence the term ‘ladder pricing’ or ‘tiered pricing’.

This paper shows that if there is heterogeneity over retail demands but not (marginal) costs, ladder pricing can achieve full channel coordination - but this is not attainable if there is any heterogeneity in cost structures across retailers. In terms of wholesale profit performance, it is shown that ladder pricing outperforms quantity discounting by an amount that tends to increase the greater the extent of heterogeneity in retailers demands *vis a vis* their costs. The optimal tariff will typically be non-linear (just as in the case of quantity discounting second degree price discrimination), although a linear ladder pricing tariff will often perform reasonably well. Interestingly, a linear ladder pricing contract (wholesale price a linear function of retail price) is shown to be formally equivalent to a revenue sharing contract (revenue sharing in conjunction with setting a uniform price per unit).

Revenue sharing contracts used in practice often have a simple linear structure, with the same form of contract being offered to all agents/retailers (see Bhattacharyya and Lafontaine, 1995). Although there is a supply chain literature on revenue sharing contracts (see e.g. Cachon and Lariviere, 2005; Bernstein, Chen and Federgruen, 2006; Gerchak and Wang, 2004; Pan *et al.*, 2010), the focus is usually on a single or fixed number of supplier/ retailer channels. To the author's knowledge, the optimization problem of designing revenue sharing or ladder pricing contracts, when downstream retailers have not only a decision on what price to set, but also on whether to participate at all, has not been addressed. In this paper, it is shown that modeling the participation decision has a significant impact on the nature of the optimal contract; the optimal contract will often feature positive revenue sharing in conjunction with a uniform price that is *below wholesale marginal cost* and quite possibly *negative* (although clearly, the overall payment from each participating retailer to the wholesaler is non-negative). Contracts in practice do not usually feature such low uniform prices, and the offering of a contract with a very low or negative uniform price might seem problematic in practice; from a PR perspective, this suggests a possible preference for presenting such contracts in a ladder pricing format.

Wholesale 'ladder pricing' is operational under the same conditions as those required for revenue sharing, and these conditions are rather more restrictive than those for wholesale second degree price discrimination. Both quantity discounting and ladder pricing contracts require that downstream retailers cannot freely arbitrage or self-supply, but ladder pricing additionally requires that prices set by individual retailers who choose to participate can be subsequently monitored by the wholesaler. Although more restrictive, revenue sharing is already manifest in a range of supply chain situations. DVD/video rentals is a classic example, but it is also manifest in transfer pricing environments and professional services (for example, letting agency fees, legal fees, accountancy fees). Professional fees often seem to be tied more to firm turnover or profitability than to the actual amount of work done. Profit or revenue sharing, alongside franchise fees, are also traditional components in franchising and licensing (see e.g. Mathewson and Winter, 1985).

The assumption that the wholesaler offers a standard ladder price contract is perhaps worth discussing further. Given the announced wholesale tariff, retailers who choose to participate and then choose a retail price in so doing reveal information about their 'type' to the wholesaler. If the wholesaler knew the type of each retailer ex ante, it could make a 'take it or leave it' bespoke deal with each retailer. The setting examined here is one in which this is not possible. That is, the wholesaler does not know the retailers type ex ante. This is not unrealistic. Even in settings where the wholesaler knows the 'name' of each retailer, the retailers may face demand uncertainty such that, period by period, they have varying 'type' (in terms of level of demand, and possibly operating cost). There may also be other considerations in play – fairness is an important consideration for humans, and a published ladder pricing contract features the same 'fairness' attribute as quantity discounting – namely that the contract on offer can be seen to be the same for all retailers who choose to participate. Finally, there is the legal and regulatory environment to consider. Offering firm specific 'bespoke' contracts to retailers is clearly more likely to encounter Robinson Patman Act litigation – something which is clearly reduced when the same contract is offered to all firms who choose to participate. The regulatory environment may also restrict firms to offering the same published tariff to all who wish to participate (as in the UK telecoms case).

From the wholesaler's perspective, the benefits of ladder pricing (and revenue sharing as a special case) over quantity discounting need to outweigh the additional monitoring costs involved, although, as a general observation, monitoring costs may tend to fall as technology advances, so the scope for ladder pricing may tend to increase over time. From a marketing perspective, it is worth noting that, precisely because this form of contract is relatively unusual, the opportunity to utilize ladder pricing may be missed – that is, it may not be considered even where it is feasible. This is certainly the case in the Telecoms case discussed above; ladder pricing has been technically feasible in Telecoms for some time, but it is only recently that it has been 'discovered' as a tariff possibility.

Making comparisons of the profit performance of quantity discounting and ladder pricing is difficult because these two forms of nonlinear pricing occur in a setting where downstream agent type space is *at least* bivariate. That is, retailers will typically have different cost structures and also face differing levels of demand. Computational difficulties are severe whenever the type and/or product space is multidimensional (a point made clearly by e.g. Wilson, 1996; Armstrong, 1996). Although significant progress has been made on mechanism design problems when both type and product space is multidimensional (see e.g. Wilson, 1999; Rochet and Stole, 2003), the solution procedures are complex and the determination of optimal tariffs generally challenging. Making a performance comparison is even more challenging because this requires not only that optimal tariffing solutions are found, but in addition, their associated profitability.

In the polar cases where the type space is univariate, it is possible to make a fairly general statement; specifically, it can be shown that if there is solely retail demand heterogeneity, ladder pricing clearly outperforms quantity discounting. By contrast, when there is solely cost heterogeneity, the tariffs have identical performance. This suggests that the intermediate case, where the type space is (at least) bivariate, this is likely to feature an intermediate level of relative performance. Section 3 of the paper illustrates this by examining a special case where retailers have constant but heterogeneous marginal costs, no fixed costs, and heterogeneous linear retail demand, with the distribution for retailer marginal costs and for demand both uniformly distributed. In this setting, the wholesaler is unable to observe individual retailer demand or retailer marginal cost but is assumed to know the joint distribution of these variables across retailers. Under these assumptions, the screening problem is well behaved and it is possible to not only compute optimal tariff solutions, but also the profit the wholesaler can earn from them. As conjectured from the results in the polar cases, it is then shown that the greater the demand heterogeneity across retailers, relative to cost heterogeneity, the better the performance of ladder pricing relative to quantity discounting. The results show that ladder pricing *can* offer significant increases in performance, and clearly suggest that this is likely to be greater to the extent that demand heterogeneity is greater than cost heterogeneity (something that seems likely in most practical applications).

2. **Relative performance of Ladder Pricing *vis a vis* Quantity Discounting**

This section discusses two polar cases (i) where there is solely retail demand heterogeneity, and (ii) where there is solely retail (marginal) cost heterogeneity. The analysis of ladder pricing solutions in these cases is reasonably straightforward, and provides a range of helpful insights regarding the nature of the solution.

2.1 Demand heterogeneity

Suppose a wholesaler sells a good to a continuum of downstream retailers who operate in exclusive territories. Retailers are parameterized via a single type variable $\theta \in [\theta_l, \theta_h]$ where $0 \le \theta_l < \theta_h$. If a retailer chooses a retail price p $(p > 0)$, it faces a level of demand $q(p,\theta)$ where $q_1(p,\theta) < 0$ (demand curves have downward slope) and $q_2(p,\theta) > 0$ (demand is strictly increasing in the type variable).[3] The wholesaler has constant marginal cost m, and retailers have the same constant marginal cost α, and there are no fixed costs. Each retailer knows its marginal cost and also the demand schedule it faces; the wholesaler is assumed to know the structure of the retailers demand functions but is unable to identify retailer type *ex ante*, and so is unable to offer bespoke contracts. It is well known that in this situation, wholesaler profitability under second degree price discrimination (quantity discounting) can improve on that under a uniform wholesale price contract, but it is inefficient in that it fails to maximise total chain profits across the population of retailers that choose to participate. For example, Robert Wilson (1997, pp. 157-8) illustrates this point in a numerical example in which demands are linear and the type distribution is uniform. By contrast, it is straightforward to show that ladder pricing yields a fully efficient solution, and one in which the wholesaler can appropriate all of this joint profit.

To see this, consider the problem of maximizing whole chain profitability. Whole chain profitability for the wholesaler and a type θ retailer is denoted π_{w+r} (subscripts '*w*' for wholesaler, '*r*' for retailer) and is given as

$$\pi_{w+r} = (p - \alpha - m)q(p,\theta) . \tag{1}$$

The choice of retail price that maximizes whole chain profit must satisfy the first order condition that

$$\partial\pi_{w+r} / \partial p = (p - \alpha - m)q_1(p,\theta) + q(p,\theta) = 0 \quad , \tag{2}$$

and the participation constraint that

$$\pi_{w+r} \ge 0 \Rightarrow p \ge \alpha + m \text{ and } q(p,\theta) \ge 0 \ . \tag{3}$$

Now suppose the wholesaler offers a wholesale price contract in which wholesale price $w(p)$ is a smooth function of retail price, and the retailers make independent decisions on what downstream price to set. The type θ retailer profit π_r is now

$$\pi_r = \left(p-\alpha-w(p)\right)q(p,\theta) \ , \tag{4}$$

and the retailer will choose *p* to satisfy the first order condition

$$\partial\pi_r / \partial p = \left(p-\alpha-w(p)\right)q_1(p,\theta)+\left(1-w'(p)\right)q(p,\theta)=0 \ , \tag{5}$$

with participation defined by

$$\pi_r \geq 0 \Rightarrow p \geq \alpha + w(p) \ \text{ and } \ q(p,\theta)\geq 0 \ . \tag{6}$$

The problem is to design a wholesale price schedule such that the solution in equations (5), (6) is identical to that in equations (2), (3). Putting (2) and (5) together, we have

$$p-\alpha-m=-\frac{q(p,\theta)}{q_1(p,\theta)}=\frac{p-\alpha-w(p)}{1-w'(p)} \tag{7}$$

so that

$$w'(p)-\frac{1}{p-\alpha-m}w(p)+\frac{m}{p-\alpha-m}=0, \tag{8}$$

an ordinary differential equation which has general solution

$$w(p)=m+\beta(p-\alpha-m) \ . \tag{9}$$

where β is an arbitrary constant. This ladder pricing contract induces exactly the same retailer price choices and participation as in the joint profit maximizing solution.

Notice that substituting the optimal ladder wholesale price function (9) into (4) gives

$$\begin{aligned}\pi_r &= \left(p-\alpha-\left\{m+\beta\left(p-\alpha-m\right)\right\}\right)q\left(p,\theta\right)\\ &=\left(1-\beta\right)\left(p-\alpha-m\right)q\left(p,\theta\right)\\ &=\left(1-\beta\right)\pi_{w+r}\end{aligned} \ , \tag{10}$$

and the profit this ladder price function yields to the wholesaler is

$$\pi_w = \left(w(p)-m\right)q = \beta\left(p-\alpha-m\right)q = \beta\pi_{w+r} \ . \tag{11}$$

Thus the optimal ladder price schedule that incentivizes all participating retailers to choose their joint-profit maximizing retail prices is formally equivalent to a profit sharing rule. This is why the ladder pricing contract works – it induces profit sharing *for all* downstream retailers, and this naturally induces them to charge the 'right' downstream price in order to maximize whole chain profitability.

Wholesaler profit is clearly then maximized by moving $\beta \to 1$, equivalent to setting a ladder price of the form $w(p) = p - \alpha$, although this pushes participation to the limit since then

$$\pi_r = \left(p-\alpha-w(p)\right)q\left(p,\theta\right) = 0 \tag{12}$$

for all retailers (that is, they are all indifferent as to whether they participate or not).[4]

This (linear) ladder pricing solution is also equivalent to a revenue sharing contract in which there is both revenue sharing alongside a uniform wholesale price. To see this, note that the overall wholesale charge under ladder pricing is

$$w(p)q = \left(m+\beta\left(p-\alpha-m\right)\right)q = \beta pq + \left((1-\beta)m - \beta\alpha\right)q, \tag{13}$$

so it is equivalent to a β share of revenue and a uniform wholesale price of $(1-\beta)m-\beta\alpha$. Notice that the uniform wholesale price is *less* than wholesale marginal cost *m* and that as the wholesaler aims to maximize profit (pushes $\beta \to 1$), this 'uniform wholesale price' is *negative* (equal to $-\alpha$, the negative of retail marginal cost). That is, the tariff that maximizes wholesaler profit depends on retailers marginal cost, but not on the wholesaler's marginal cost.

Revenue sharing arrangements observed in practice (e.g. in video rentals, in franchising) often feature uniform pricing in addition to a sharing of revenue. However, these contracts typically feature revenue sharing in conjunction with a positive uniform wholesale price either equal to or above *wholesaler* marginal cost. Such a tariff (revenue share plus positive wholesale uniform price) is clearly not profit maximizing for the wholesaler, nor is it efficient from the perspective of the whole chain. This observation that an optimal revenue sharing contract should feature below (wholesale) marginal cost pricing seems to be an interesting observation in its own right. From a practical perspective, if it was in fact desirable to offer a revenue sharing contract with a negative uniform price, given this might appear somewhat unusual and perhaps confusing to retailers, it would seem that presenting the contract in a ladder pricing format has much to recommend it.

Given ladder pricing/revenue sharing achieves maximum whole chain profitability, clearly it will outperform an optimized quantity discounting contract. To illustrate the difference in relative performance, it is useful to give a simple numerical example. Suppose then retailers have linear demands differentiated by a single 'type' variable θ which defines an inverse demand function of the form[5]

$$p = \theta - q, \tag{14}$$

where p is the retail price chosen by the retailer (assumed to be profit maximizing uniform price) and q is the amount purchased from the wholesaler and then sold on to the final retail market. The distribution for the retailer demand intercept is assumed uniform on an interval $[\theta_l, \theta_h]$ $(0 \leq \theta_l < \theta_h)$. The wholesaler has constant marginal cost m, and retailers all have the same constant marginal cost α, and there are no fixed costs. Each retailer knows its marginal cost and also the demand schedule it faces; the wholesaler is unable to distinguish these characteristics *ex ante*, and so is unable to offer bespoke contracts.

The quantity discounting solution to this special case of the above model has been discussed by Robert Wilson (1997, pp. 157-8); Wilson shows that if retailers all have the same marginal cost $\alpha \geq 0$ (and this is common knowledge), and the wholesaler can select an arbitrary outlay schedule, the optimal schedule involves quantity discounting such that marginal price declines linearly with the quantity retailers choose to purchase. Specifically, the optimal marginal price $w^*(q)$ offered to all retailers takes the form

$$w^*(q) = \tfrac{1}{2}(\theta_h + m - \alpha) - q. \tag{15}$$

Notice that not all retailers will participate; only those with type θ satisfying

$$\theta \geq \tfrac{1}{2}(\theta_h + m - \alpha) \tag{16}$$

do so. The Wilson example is actually a special case in which in which the parameters take values $m = \alpha = 0, \theta_l = 0, \theta_h = 1$ (a benchmark case), and Wilson shows that the maximum attainable wholesaler profit, under quantity discounting is

$$\Pi = \tfrac{1}{24} = 0.041666 \tag{17}$$

with only half the retailers participating (those with type $\theta \geq \frac{1}{2}$). By contrast, under the ladder pricing solution, when $m = \alpha = 0, \theta_l = 0, \theta_h = 1$, the optimal solution is $w(p) = p$, there is no exclusion ($\theta_e = 0$), and profit to the wholesaler is

$$\Pi_W = \int_0^1 \left(w(p) - m\right) q(p) d\theta = \int_0^1 \left(\tfrac{1}{2}\theta\right)^2 d\theta = \tfrac{1}{12} = 0.0833. \qquad (18)$$

Thus the maximum wholesale profit under ladder pricing is exactly twice that which can be earned from quantity discounting (actually, the profit earned is exactly twice that under quantity discounting whatever the level of retailer marginal cost).

The fact that optimal ladder pricing is equivalent to profit (or revenue) sharing in the case where all retailers have identical cost functions, with constant marginal costs, should not be surprising. Unlike with second degree price discrimination, ladder pricing in this case is equivalent to profit sharing and as previously remarked, this enables the wholesaler is able to incentivize *all* retailers to choose downstream prices so as to maximize whole chain profitability whilst at the same time the wholesaler takes all of this whole chain profitability.

2.2 *Cost Heterogeneity*

In the other polar case, where all retailers face the same demand function, but have heterogeneous marginal costs, the maximum profit performance for the wholesaler is the same whether a quantity discounting outlay schedule or a ladder pricing contract is offered. This follows because, in this case (in contrast to the demand heterogeneity case), there is a one-to-one mapping between prices and quantities. To see this, suppose marginal retailer cost is α, where this retailer type variable α has an arbitrary distribution over support $\left[\alpha_l, \alpha_h\right]$ ($0 \leq \alpha_l < \alpha_h$), whilst the retailer demand type variable θ is constant across retailers. The profit a type α retailer earns when faced with the ladder pricing contract is given by (4) as $\pi_r = \left(p - \alpha\right) q(p) - w(p) q(p)$ (omitting the demand type parameter θ since it is now constant across retailers), whilst for the quantity discounting case, if the wholesaler offers a revenue outlay schedule $R(q)$ (with marginal price $\rho(q) = R'(q)$), the type α retailer profit is $\pi_r = \left(p - \alpha\right) q(p) - R\left(q(p)\right)$. These profit functions are identical if

$$R\left(q(p)\right) = w(p) q(p) \; . \qquad (19)$$

That is, given the structure of $q(p)$ is common knowledge, if $w(p)$ was an optimal ladder pricing contract, then this is reproduced by a quantity discounting outlay schedule $R(.)$ defined by (19), and equally, if $R(.)$ denoted a profit maximizing choice for the revenue outlay schedule, this can be reproduced via the ladder pricing contract defined as $w(p) = R\left(q(p)\right)/q(p)$.[6] Intuitively, the fact that ladder pricing and quantity discounting give the same performance in this case is to be expected; where private information solely concerns retailer marginal cost, the wholesaler is no longer able to incentivize joint profit maximizing behavior for retailers, and the optimal ladder pricing solution is precisely equivalent to that of optimal quantity discounting (because, as previously remarked, all retailers have the same demand function in this special case, there is a one-to-one mapping between prices and quantities via this single demand function). By contrast, when all the heterogeneity lies in retail demands (all retailers having the same marginal cost), ladder pricing is more effective at extracting profit because, unlike quantity discounting, it can be designed to provide incentives for *all* retailers to maximize whole chain profitability.

This observation is suggestive of the idea that the relative performance of the two contracts may depend on the relative extent of demand and cost heterogeneity – with the greater the former, the better the relative performance of ladder pricing *vis a vis* quantity discounting. This hypothesis is explored in the next section.

2.3 Cost and Demand heterogeneity

In this section, the case where there is both demand and cost heterogeneity is examined. As previously remarked, it is examined in a simplified setting involving linear demand as in equation (14), where θ denotes the upper intercept of the inverse retailer demand curve, distributed uniformly on $\theta \in [0, \theta_h]$ (where $\theta_h > 0$) whilst retailer marginal cost α is distributed uniformly on the range $\alpha \in [0,1]$. In this way, the bivariate uniform distribution on the rectangle $[0,1] \times [0, \theta_h]$, becomes a function solely of the parameter θ_h. As $\theta_h \to 0$, ladder pricing and quantity discounting solutions can be expected to asymptote to the polar case where there is purely retailer cost heterogeneity, whilst as $\theta_h \to \infty$, the solutions should asymptote toward the case where there is purely retail demand heterogeneity. Thus the expectation is that wholesale profitability under ladder pricing will be closer to that for quantity discounting for low θ_h, but that, as θ_h is increased (as demand heterogeneity becomes dominant relative to cost heterogeneity), so ladder pricing performance will improve toward the level obtained in section 2.1. To reduce algebraic clutter, wholesaler marginal cost is set at m=0 at the outset. Given the rectangular uniform distribution, the optimal quantity discounting outlay schedule can be shown to be piecewise linear. One would expect a similar result for the ladder pricing, with a linear schedule being close to optimal for larger θ_h. Given the complexity of obtaining a general non-linear ladder pricing solution, for relative simplicity, only the class of linear schedules is considered for ladder pricing. This provides a lower bound for profit performance for ladder pricing, and hence it suffices to show the likely extent of out-performance. It also has merit in that it provides a direct comparison of optimal revenue sharing with optimal non-linear quantity discounting in the presence of both demand and cost heterogeneity.

The Ladder Pricing Solution

Ladder price is assumed linear and increasing in retail price, taking the form

$$w(p) = \gamma_0 + \gamma_1 p. \tag{20}$$

where $\gamma_1 \in [0,1]$ and γ_0 is unrestricted in sign. The problem is to maximize wholesaler profit by selecting γ_0, γ_1. A participating retailer earns profit

$$\begin{aligned} \pi_r &= (p-\alpha-w)q = (p-\alpha-w)\left(\theta-p\right) \\ &= (p-\alpha-\gamma_0-\gamma_1 p)\left(\theta-p\right) \geq 0 \end{aligned} \tag{21}$$

and the retailer *FONC* is that

$$\begin{aligned} &\partial\pi_r / \partial p = -(p-\alpha-\gamma_0-\gamma_1 p) + (1-\gamma_1)\left(\theta-p\right) = 0 \\ &\Rightarrow -\left((1-\gamma_1)p-\alpha-\gamma_0\right) + (1-\gamma_1)\left(\theta-p\right) = 0 \\ &\Rightarrow p = \frac{\alpha+\gamma_0+(1-\gamma_1)\theta}{2(1-\gamma_1)} = \tfrac{1}{2}\theta + \tfrac{1}{2}\frac{\alpha+\gamma_0}{(1-\gamma_1)}. \end{aligned} \tag{22}$$

Excluded retailers are those that cannot earn positive profit for any choice of q; marginal retailers are those who are only able to earn zero profit at most. Denote a marginal retailer as one with parameter values θ_e, α_e who chooses quantity q_e and sets retail price p_e. Zero profit means that

$$\pi_r = \left((1-\gamma_1)\,p_e - \alpha_e - \gamma_0\right)\left(\theta_e - p_e\right) = 0, \tag{23}$$

and participation means that the retailer *FONC* must hold; thus from (22),

$$p_e = \tfrac{1}{2}\theta_e + \tfrac{1}{2}\frac{\alpha_e + \gamma_0}{(1-\gamma_1)}. \tag{24}$$

Using (23) and (24), a little algebra establishes that the solution is uniquely that

$$\theta_e = (\alpha_e + \gamma_0)/(1-\gamma_1) \Rightarrow \alpha_e = (1-\gamma_1)\theta_e - \gamma_0 \,, \tag{25}$$

and that

$$p_e = \theta_e . \tag{26}$$

This means there is a set of marginal participating retailers, defined as

$$\left\{(\alpha_e, \theta_e) : \alpha_e \in [0,1], \theta_e \in [0, \theta_h], \alpha_e = (1-\gamma_1)\theta_e - \gamma_0\right\}$$

and the set of all who participate, denoted Ω, conditional on the choices of γ_0, γ_1, is

$$\Omega = \left\{(\alpha, \theta) : \alpha \in [0,1], \theta \in [0, \theta_h], \alpha \le (1-\gamma_1)\theta - \gamma_0\right\} \;. \tag{27}$$

The shape of the participation set depends on the wholesaler's choice of ladder price schedule (the values for γ_0, γ_1) as illustrated in figures 1 and 2.

Figure 1 – the Participation set Ω when $(1-\gamma_1)\theta_h \le 1+\gamma_0$ (or $\theta_h \le \theta_1$)

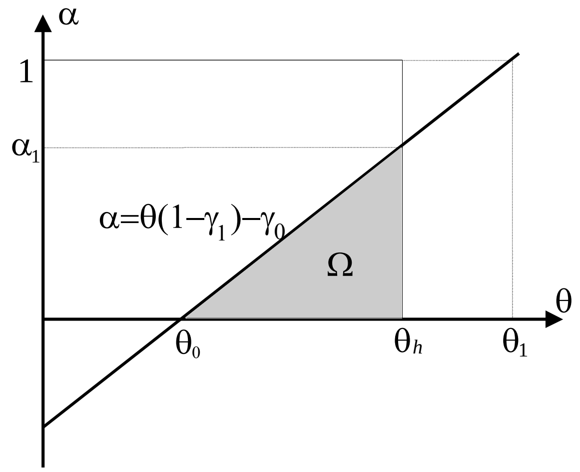
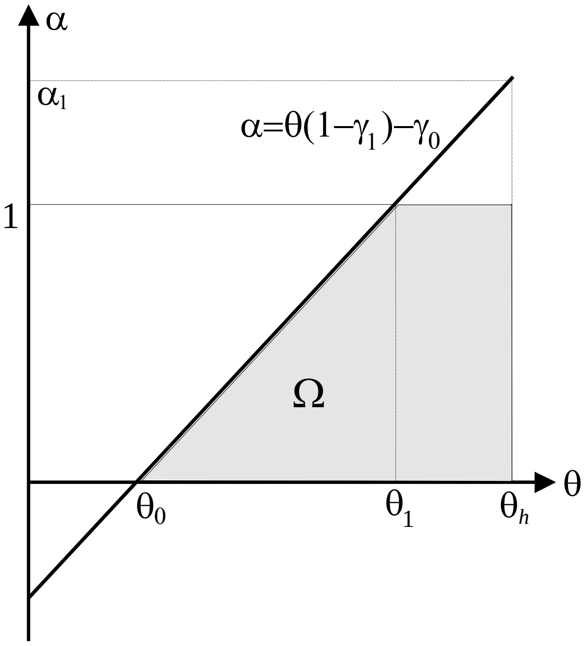


Figure 2 – the Participation set Ω when $(1-\gamma_1)\theta_h \ge 1+\gamma_0$ (or $\theta_h \ge \theta_1$)



The participation region needs some careful handling - it turns on the following two key parameters:

$$\theta_0 = \gamma_0 / (1-\gamma_1) \tag{28}$$

$$\theta_1 = (1+\gamma_0)/(1-\gamma_1) \tag{29}$$

These values are shown in figures 1 and 2, which show how the participation region depends on the choice of γ_1, γ_2. The graph of $\alpha = (1-\gamma_1)\theta - \gamma_0$ (in figure 1 for $\theta \in [\theta_0, \theta_h]$, in figure 2, for $\theta \in [\theta_0, \theta_1]$) represents the set of marginal retailers; figure 1 illustrates the case where $\theta_h \le \theta_1$, in which case the participation region is triangular, whilst figure 2 gives the case where $\theta_h > \theta_1$, where it is a trapezoid. It is possible that θ_0 is negative (since γ_0 is unrestricted in sign). This will affect the evaluation of wholesale profit as the participation region is then truncated to the left at $\theta = 0$ in both figures. This is straightforward to take into account (see below).

Wholesaler profit Π_W depends on whether figure 1 or figure 2 applies, and also whether θ_0 is negative or positive, as follows:

<u>Case (a)</u> If $\theta_h \le \theta_1 = (1+\gamma_0)/(1-\gamma_1)$: (as in figure 1)

Then

$$\Pi_W = \int_\Omega w(p)q(p)\left(1/\theta_h\right)d\theta d\alpha = \left(1/\theta_h\right)\int_\Omega\left(\gamma_0+\gamma_1 p\right)\left(\theta-p\right)d\theta d\alpha$$
$$=\left(1/\theta_h\right)\int_{Max(\theta_0,0)}^{\theta_h}\int_0^{\theta(1-\gamma_1)-\gamma_0}\left(\gamma_0+\gamma_1 p\right)\left(\theta-p\right)d\alpha d\theta$$
$$=\left(1/\theta_h\right)\int_{Max(\theta_0,0)}^{\theta_h}\int_0^{\theta(1-\gamma_1)-\gamma_0}\left(\gamma_0+\gamma_1\left(\tfrac{1}{2}\theta+\tfrac{1}{2}\frac{\alpha+\gamma_0}{(1-\gamma_1)}\right)\right)\left(\theta-\tfrac{1}{2}\theta-\tfrac{1}{2}\frac{\alpha+\gamma_0}{(1-\gamma_1)}\right)d\alpha d\theta. \tag{30}$$

Notice the lower limit for θ-integration is $Max(\theta_0,0)$ and that (22) is used to replace p in line 3). Expanding this and performing the integrations, this can eventually (after a lot of routine algebra) be reduced to

$$\Pi_W = \tfrac{1}{4\theta_h}\left[\begin{array}{l}\left(\tfrac{2}{3}(1-\gamma_1)\gamma_0-\tfrac{1}{3}\gamma_0\gamma_1\right)\theta^3+\tfrac{1}{4}\gamma_1(1-\gamma_1)\theta^4 \\ -\left(\tfrac{1}{2}\gamma_0{}^2\frac{(2-\gamma_1)}{(1-\gamma_1)}+\gamma_0{}^2\right)\theta^2+\gamma_0{}^3\frac{(2-\gamma_1)}{(1-\gamma_1)^2}\theta\end{array}\right]_{Max(\theta_0,0)}^{\theta_h} \tag{31}$$
$$-\frac{1}{12\theta_h(1-\gamma_1)^3}\left[\gamma_0\theta^3+\tfrac{1}{4}\gamma_1\theta^4\right]_{Max(\theta_0,0)(1-\gamma_1)-\gamma_0}^{\theta_h(1-\gamma_1)-\gamma_0} .$$

(Note in these equations the notation $\left[f(x)\right]_a^b = f(b)-f(a)$). This is a function of the parameters γ_0,γ_1 given that θ_0,θ_1 are determined by these parameters, and so it can be optimized numerically.

<u>Case (b)</u> $\theta_h > (1+\gamma_0)/(1-\gamma_1)$ (as in Figure 2)

Then

$$\Pi_W = \int_\Omega w(p)q(p)\left(1/\theta_h\right)d\theta d\alpha = \left(1/\theta_h\right)\int_\Omega\left(\gamma_0+\gamma_1 p\right)\left(\theta-p\right)d\theta d\alpha$$
$$=\left(1/\theta_h\right)\left[\int_{Max(\theta_0,0)}^{\theta_1}\int_0^{\theta(1-\gamma_1)-\gamma_0}\left(\gamma_0+\gamma_1 p\right)\left(\theta-p\right)d\alpha d\theta+\int_{\theta_1}^{\theta_h}\int_0^1\left(\gamma_0+\gamma_1 p\right)\left(\theta-p\right)d\alpha d\theta\right]$$
$$=\left(1/\theta_h\right)\left[\begin{array}{l}\int_{Max(\theta_0,0)}^{\theta_1}\int_0^{\theta(1-\gamma_1)-\gamma_0}\tfrac{1}{4}\left(2\gamma_0+\gamma_1\theta+\gamma_1\frac{\alpha+\gamma_0}{(1-\gamma_1)}\right)\left(\theta-\frac{\alpha+\gamma_0}{(1-\gamma_1)}\right)d\alpha d\theta \\ +\int_{Max(\theta_0,0)}^{\theta_h}\int_0^1\tfrac{1}{4}\left(2\gamma_0+\gamma_1\theta+\gamma_1\frac{\alpha+\gamma_0}{(1-\gamma_1)}\right)\left(\theta-\frac{\alpha+\gamma_0}{(1-\gamma_1)}\right)d\alpha d\theta\end{array}\right]. \tag{32}$$

which again can be eventually be simplified to obtain

$$\Pi_W = (1/\theta_h)\left(\begin{array}{c} \frac{1}{4}\left[\begin{array}{l} \left(\frac{2}{3}(1-\gamma_1)\gamma_0 - \frac{1}{3}\gamma_0\gamma_1\right)\theta^3 + \frac{1}{4}\gamma_1(1-\gamma_1)\theta^4 \\ -\left(\frac{1}{2}\gamma_0{}^2\frac{(2-\gamma_1)}{(1-\gamma_1)} + \gamma_0{}^2\right)\theta^2 + \gamma_0{}^3\frac{(2-\gamma_1)}{(1-\gamma_1)^2}\theta \end{array} \right]_{Max(\theta_0,0)}^{\theta_1} \\ -\frac{1}{12}\frac{1}{(1-\gamma_1)^3}\left[\gamma_0\theta^3 + \frac{1}{4}\gamma_1\theta^4\right]_{Max(\theta_0,0)(1-\gamma_1)-\gamma_0}^{\theta_1(1-\gamma_1)-\gamma_0} \\ +\frac{1}{4}\left[\gamma_0\theta^2 + \frac{1}{3}\gamma_1\theta^3 - \frac{\gamma_0{}^2(2-\gamma_1)+\gamma_0+\frac{1}{3}\gamma_1}{(1-\gamma_1)^2}\theta\right]_{\theta_1}^{\theta_h} \end{array} \right) \qquad (33)$$

Again this is a function of the parameters γ_0, γ_1 and can be numerically optimized. Results are presented in section 2.3 below, where they are compared with those obtained for the quantity discounting solution.

The quantity discounting solution

The optimal solution can be obtained using the demand profile approach (Wilson, 1993). Care needs to be taken to deal appropriately with the participation region, as in the case above for ladder pricing. Because the analysis is both lengthy and intricate to present it is omitted.[7] The profit maximizing wholesale quantity discounting outlay schedule that results is:

$$w^*(q) = \tfrac{1}{2}\left(\theta_h - \tfrac{1}{2}\right) - q \qquad q \in [0, \tfrac{1}{2}\theta_h - \tfrac{3}{4})$$

$$w^*(q) = \tfrac{1}{3}\theta_h - \tfrac{2}{3}q \qquad q \in [\tfrac{1}{2}\theta_h - \tfrac{3}{4}, \tfrac{1}{2}\theta_h]\ .$$

The associated wholesale profit function is then a polynomial function of θ_h ; it takes the form

<u>Case (a)</u> If $\theta_h \geq \frac{3}{2}$: Wholesaler profit Π_W is given by

$$\begin{aligned} \theta_h\Pi_W &= \left(-\tfrac{1}{2}\left(\theta_h - \tfrac{1}{2} - \left(z_e * - \tfrac{1}{2}\theta_h + \tfrac{1}{4}\right)\right)\left(z_e * - \tfrac{1}{2}\theta_h + \tfrac{1}{4}\right)\right)\left(\theta_h - 1 - z_e *\right) \\ &+ \tfrac{1}{2}\left(-\tfrac{1}{2}\left(\theta_h - \tfrac{1}{2} - \left(z_e * - \tfrac{1}{2}\theta_h + \tfrac{1}{4}\right)\right)\left(z_e * - \tfrac{1}{2}\theta_h + \tfrac{1}{4}\right) + \left(\tfrac{1}{24}\theta_h{}^2 - \tfrac{1}{8}\theta_h + \tfrac{3}{32}\right)\right) \\ &+ \tfrac{1}{32}\int_{z_e*}^{\theta_h - 1}\left(\left(-12\theta_h{}^2 + 12\theta_h - 3\right) + 16\left(2\theta_h - 1\right)z - 16z^2\right)dz \\ &+ \tfrac{1}{48}\int_{\theta_h - 1}^{\theta_h}\left(-5\theta_h{}^3 + 23\theta_h{}^2 z - 27\theta_h z^2 + 9z^3\right)dz \end{aligned}$$

where

$$z_e{*} = \tfrac{1}{2}\left(\theta_h - \tfrac{1}{2}\right).$$

Case (b) If $\theta_h < \tfrac{3}{2}$: Wholesaler profit Π_W is given by

$$\Pi_W = \tfrac{1}{48\theta_h}\left(\alpha_0 z + \tfrac{1}{2}\alpha_1 z^2 + \tfrac{1}{3}\alpha_2 z^3 + \tfrac{9}{4} z^4\right)_{z_e{*}}^{\theta_h}$$

where

$$\alpha_0 = -3\theta_h^{\ 3} - 12\theta_h^{\ 2} z_e + 18\theta_h z_e^{\ 2}$$
$$\alpha_1 = 21\theta_h^{\ 2} + 12\theta_h z_e - 18 z_e^{\ 2}$$
$$\alpha_2 = -27\theta_h$$

and

$$z_e{*} = \tfrac{1}{3}\theta_h .$$

It is therefore straightforward to evaluate profit performance as a function of the parameter θ_h .

Numerical Results

Having obtained the solutions, this section reports results for ladder pricing and quantity discounting; Table 1 below presents these results for a range of values for θ_h; it reveals that ladder pricing always outperforms quantity discounting in this setting, and that the difference in profitability is strictly increasing in θ_h. The relative profitability is also broadly increasing with θ_h.

Relative profitability is relatively static until θ_h exceeds 1.5. After that it climbs quickly toward an asymptote of around 200%. This is consistent with the results reported in section 2.1; with all the heterogeneity concentrated on retailer marginal cost, the profit ratio was 100% and when all the heterogeneity was concentrated on retailer demand the profit ratio was 200%.

Table 1 Wholesaler Profitability

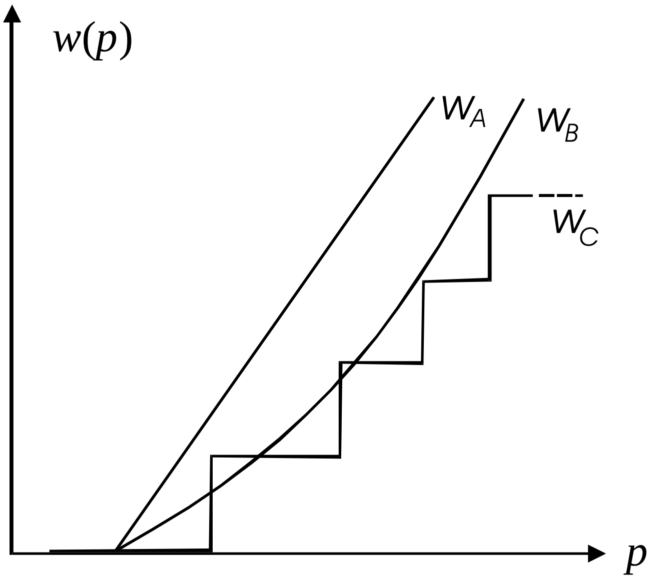
θ_h	γ_0 *	γ_1 *	$\Pi_W^*(LP)$ under Ladder Pricing	$\Pi_W^*(QD)$ under Quantity Discounting	Profit Difference: $\Pi_W^*(LP)$ minus $\Pi_W^*(QD)$	% Ratio $\frac{\Pi_W^*(LP)}{\Pi_W^*(QD)}$
0.1	0.000	0.500	1.04E-05	9.26E-06	1.14E-06	1.123
0.5	0.000	0.500	1.30E-03	1.16E-03	1.43E-04	1.123
1	0.000	0.500	0.010	9.26E-03	7.41E-04	1.080
1.5	0.000	0.500	0.035	3.13E-02	3.75E-03	1.120
2	0.000	0.500	0.083	0.073	0.010	1.138
2.5	-0.025	0.545	0.161	0.135	0.026	1.189
3	-0.055	0.595	0.271	0.219	0.052	1.239
5	-0.140	0.715	1.057	0.760	0.297	1.390
10	-0.234	0.825	5.622	3.573	2.049	1.574
20	-0.304	0.891	26.47	15.448	11.022	1.713
50	-0.367	0.942	185.61	101.073	84.537	1.836
100	-0.400	0.963	777.6	410.448	367.152	1.895
500	-0.446	0.987	20385.7	10385.448	10000.252	1.963
1000	-0.459	0.992	82230.7	41604.198	40626.502	1.977

Notice that for small θ_h, the optimal ladder price function is simply $w(p) = 0.5p$, and this coincides with that for the polar case where there is all cost heterogeneity and no demand heterogeneity. As the extent of demand heterogeneity relative to cost heterogeneity increases (as θ_h increases), so γ_1* converges to the value of unity obtained in the other polar case, in section 2.1, where there was solely demand heterogeneity. In that polar case, the solution was $w(p) = p - \alpha$ (where α, the retailer marginal cost was in that case a unique number, being the same for all retailers). In this section, the marginal cost for retailers is distributed over the range [0,1], so the average marginal cost across all retailers is 0.5. Of course, the average marginal cost of *participating* retailers will be less than 0.5 because, *ceteris paribus*, the higher the marginal cost, the more likely the retailer is priced out of the market. In fact, computing the participation percentage, this is strictly increasing and rises monotonically toward 100% as θ_h is increased – so the average marginal cost is in fact converging on 0.5. Hence it is plausible that, as revealed in Table 2, γ_0 * should converge toward a figure of minus 0.5 as per the polar case.

3. Ladder Pricing Step Functions

The recent practice of ladder pricing in UK Telecoms has sometimes featured linear tariffs, but more commonly, step functions. Figure 3 below illustrates linear ladder pricing (w_A) non-linear ladder pricing (w_B) and step function ladder pricing (w_C) each as functions of the retail price *p*. The first practical observation to make is that step functions (such as w_C in figure 3) can be viewed as approximations to continuous functions – and naturally a step function can be designed as an increasingly close approximation to any given continuous smooth wholesale price function, the smaller the step size chosen. The first forms used in practice actually featured step functions, so it is of some interest to discuss the pros and cons of this *vis a vis* the other forms of contract. Practitioners have argued that step functions make the practice of ladder pricing operationally easier. This turns on the question of how accurately the wholesaler is able to monitor retail prices. In the UK Telecom case, downstream retailers were mobile network operators operating relatively complex forms of retail pricing (menus of multi-part tariffs for example). The wholesale price in these cases was designed to be functionally related to the *average* downstream price. Given the complexity of the downstream offerings, and given likely variations over time in quantities sold, practitioners tended to argue that the exact average downstream price might fluctuate somewhat and that there would be less need to adjust the wholesale price continuously over time if ladder pricing took the form of a step function (simply because even though it might fluctuate, the average price is more likely to locate on a single step, such that the wholesale price does not vary). It is clear that, for a given wholesale pricing step function, on a step, wholesale price is constant, so the 'double marginalization' incentive occurs (Spengler, 1950). That is, for alternative choices of retail price that induce the same wholesale price, it is as if the retailer is facing a uniform price. Naturally, this also implies retailers will tend to want to pitch their price nearer toward the foot of a 'step up' on the wholesale price function – although they still have the question of which step to choose to locate on.

Figure 3 – Ladder Pricing Contracts



4. Legal and Regulatory Considerations

Wholesalers who price discriminate as between different retailers can fall foul of the Robinson Patman Act in the US. The fact that the same contract is offered to all who choose to participate might be thought to guarantee 'fairness' but this is by no means clear; Moorthy (1987) points out that quantity discount schedules, even if the same contract is offered to all retailers, will generally mean that different retailers are charged different prices (both marginal and average prices) contingent on the amount they choose to purchase – and that this can be grounds for claiming injury in that firms competition with other retailers.[8] It might seem that similar arguments apply to the ladder pricing case; however, the argument may be harder to make in this case. The point is that, insofar as retailers are in competition downstream, and insofar as they have similar products, they will need to charge similar prices, they will face similar wholesale charges. However, insofar as retail products are heterogeneous, retailers may choose different retail prices and hence attract different wholesale prices (that are not related to the underlying cost of wholesale supply), hence leading to possible action under the act. In this context, it is interesting to note the parallels between ladder pricing and revenue sharing, in that the special case of a linear ladder pricing schedule is in fact equivalent to revenue sharing plus a uniform unit wholesale price – so the legal case is pretty much the same for both these types of channel arrangement.[9]

Whilst in the US, Telecoms are regulated with a relatively light touch, this is not the case in the UK and Europe, where regulators quite commonly get involved in highly complex economic assessments of Telecom practice. In the case of ladder wholesale pricing, Ofcom, the UK regulator has been involved in a dispute between wholesale operators and mobile network operators concerning wholesale ladder pricing proposals by the former companies (see Ofcom, 2010, 2013, for example). In these cases, no issue was raised as to whether the proposed charges represented an abuse of a dominant position, and Ofcom considered that it *could* be fair and reasonable for the wholesaler to introduce tiered wholesale charges. Ofcom's principal concern was whether the proposed charges would be likely to benefit mobile network customers through inducing mobile network operators to reduce their retail charges for calls to these non-geographic numbers (since in general, Ofcom took the view that current retail prices were too high). Ofcom's initial view was that the wholesale tariffs did not guarantee consumer benefits, and so ruled against allowing the tariffs. The wholesaler appealed, and the case was taken before the Competition Appeals Tribunal, where the Ofcom judgment was overturned. The case was then taken by the MNOs to the Court of Appeal, which overturned the CAT judgment, and then further appealed by the network operator before the Supreme Court, which finally ruled (July, 2014) that such tariffs were indeed permissible and that the onus was on those who objected to show the probability that the imposition of such tariffs would result in material harm to callers. Thus, wholesale ladder pricing is back on the agenda for Telecom fixed line network operators.

5. **Conclusions**

In this paper, a new variant on wholesale price discrimination has been discussed. 'Ladder' or 'tiered' pricing involves linking the wholesale price to the price chosen downstream by retailers, generally by setting a higher wholesale price, the higher the choice of retail price downstream. Although revenue sharing contracts have found wide use in a range of settings, the generalization to non-linear ladder pricing appears to be new. Recent practice in UK Telecom markets appears to be the only example of non-linear ladder pricing in action (of which the author is aware); in this case, several wholesale service providers have offered a non-linear step function wholesale tariff and others have offered a linear or piecewise linear tariff. It seems useful to highlight the possibility and potential for introducing such tariffs since, whenever revenue sharing is feasible, so too is the more general form of (non-linear) ladder pricing.

When a wholesale ladder pricing or revenue sharing contract is offered to all who choose to participate, standard forms of revenue sharing contract (in which a uniform price is set at or above wholesaler marginal cost) are unlikely to be optimal. Profit maximizing ladder pricing, when the ladder is restricted to the linear form, is equivalent to a revenue sharing contract in which revenue sharing is combined with setting a below wholesaler marginal cost and potentially negative uniform price (whilst of course the overall payment to the wholesaler remains positive for all retailers who choose to participate). In particular, the greater the retailer demand heterogeneity relative to cost heterogeneity, the lower this implied uniform price should be in a revenue sharing contract. As a practical concern, revenue sharing contracts featuring a negative uniform price might appear less than attractive from a PR perspective as it is harder to 'explain' or rationalize. That is, when the wholesaler sets a positive uniform price (in conjunction with revenue sharing), the uniform price can be 'justified' to retailers because it can be related to wholesale production costs. By contrast, if the wholesaler sets a revenue sharing contract in which there is a negative uniform price, this clearly is 'less easy' to 'cost justify'. No such problem arises for the equivalent ladder pricing contract – setting such a pricing contract thus neatly sidesteps this practical concern.

It was also shown that, within the model, where there is heterogeneity in both retailer demands and costs, so long as there is *some* demand side heterogeneity, profit maximizing ladder pricing always outperforms quantity discounting. Further, *ceteris paribus*, the relative performance of ladder pricing *vis a vis* quantity discounting (broadly) tends to increase with the extent of demand heterogeneity relative to marginal cost heterogeneity. Whilst this insight seems likely to hold in more general setting, this remains to be demonstrated (unfortunately, making performance comparisons when there are non-linear demands, non-linear cost functions, more general forms for the type distribution function etc. is seriously challenging).

The range of applications for wholesale ladder pricing is more restrictive than that for quantity discounting. All the conditions required for the latter must be present, and in addition, it must be possible for the wholesaler to monitor prices set at the individual retailer level. However, whilst the set of applications for which ladder pricing is feasible is more restricted, it remains a significant set, since revenue sharing is practiced in various environments (notably DVD/video rentals/ transfer pricing/franchising/licensing) and ladder pricing has recently been used in UK telecom markets.

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Endnotes

[1] A supply chain is said to be coordinated if contract arrangements lead to joint profits being maximised.

[2] The case first came before the Competition Appeals Tribunal, who concluded that there may well be incentives to reduce average retail price, notwithstanding the fact that complex forms of retail price discrimination are practiced in these markets (network operators offering a range of tariff 'menus'). Competition Appeals Tribunal Case Numbers 1168/3/3/10,1169/3/3/10,1151/3/3/10. Transcripts, including the final judgement, are available from the CAT website: http://www.catribunal.org.uk/ Economic analysis of the incentive to reduce retail price, and on associated welfare consequences, can be found in reports presented as annexes to the Ofcom (2010, 2013) publications.

[3] Notation: $q_1(p,\theta) \equiv \partial q(p,\theta)/\partial p; \quad q_2(p,\theta) \equiv \partial q(p,\theta)/\partial \theta$ etc.

[4] For (motivational) reasons that lie outside the model, there will no doubt be trade off, and it will rarely be optimal to push β 'too high'.

[5] The slope coefficient can be normalised to -1 by suitable choice of units.

[6] This equivalence can be illustrated numerically if one makes more specific assumptions about the demand function and the distribution for retailer marginal cost (for example, with linear demand and a uniform distribution for retailer marginal cost – a derivation for this case is omitted, given that it is a limiting result for the bivariate distribution case examined in section 3 below).

[7] A full derivation of the solution for this case is available at the author's website https://www.staff.ncl.ac.uk/i.m.dobbs/Pages/Research.htm.

[8] For example small retailers, who purchase less, will be paying higher prices for the wholesale product.

[9] That is, any practical step function wholesale pricing schedule can be viewed as similar to a form of revenue sharing contract.